

Nash ICM Library FAQ

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1 Basic Algorithm: Fictitious Play

For a good in-detail description of *Fictitious Play*, please refer to *The Mathematics of Poker* by Chen/Ankenman. The basic idea is quite simple: Players are iteratively adjusting their strategies, based on their opponents strategy of the last iteration. If the process converges, we found a *Nash Equilibrium* (In a *NE* no player can unilaterally adjust his strategy to achieve a higher EV).

//TODO: more detailed fp description

However, there is no guarantee that this process will eventually converge.

2 Implementation Restrictions

To allow efficient calculation of the *Fictitious Play* iterations, following restrictions are enforced:

- To eliminate the possibility of postflop actions, players can only push or fold pre flop. No other actions are allowed.
- At most 3 players can enter the pot. After 3 players entered the pot, all remaining players are auto-folded.¹
- During the fictitious play calculation, players are restricted to use predefined handrankings for pushing and calling.
- Card removal effects due to folding are not considered. This means the starting hand distribution of a player will be assumed to be perfectly random if all players folded so far. However, card removal effects due to pushes are correctly considered.

¹Even if a player in the blinds is all-in by posting the blind, the player will be considered folded if 3 players are in the pot before the actions advances to him.

2.1 Handrankings

During the *Fictitious Play* calculation, all ranges are handranking-restricted. For pushing and calling/overcalling, separate handrankings are defined. A handranking defines an order upon starting hands, like $AA > KK > \dots > 72o$. If a handranking defines $H_1 > H_2$, a player is not allowed to play hand H_2 , unless H_1 is also played. That means we are restricting the number of pure strategies to 169^2 , instead of 2^{169} in the unrestricted case.

These restrictions apply during the *Fictitious Play* calculation only. After the *Fictitious Play* calculation is done, the hand selection is optimized. During this process we will replace the range with the most profitable starting hands, assuming all opponents play their *Fictitious Play* ranges, while leaving the fraction of hands played unchanged. e.g. If a player's *Fictitious Play* range is 20% in a given spot, the optimized range will also consist of approximately 20% - just the actual hand selection will change to maximise his EV.

2.2 Convergence and Result Properties

Generally, the *Fictitious Play* process is not guaranteed to converge. And, in fact, it is very rare to find a true *Nash Equilibrium* for any hands with more than 3 players. A decreasing weighting is used during the *Fictitious Play* calculation, to stabilize results. The resulting ranges are basically an approximation of a *Nash Equilibrium*.

This means that the *Nash Equilibrium* property will usually not hold. Players will be able to change their ranges and exploit the calculated ranges. Ranges will usually include either a few slightly -EV hands, or exclude a few slightly +EV hands. However, the EV gained by varying from the calculated results is very small in most cases. For all practical regards, the results can be treated like true *Nash Equilibria*. It will be extremely hard for opponents to significantly exploit the calculated ranges while sticking to the push-or-fold restriction.

²Folding the top hand of a handranking (AA) is not allowed, therefore 169 possible ranges.

3 Some common Misconceptions/Mistakes

3.1 Opponents mistakes are NOT always +EV to us

This point is extremely important, please make sure you understand the following concept. The calculated push ranges are hard to exploit, in the sense that your opponent can not improve his own *EV* significantly by deviating from the calculated call range. However, in a typical payout structure³ this does **not** imply that you will gain *EV* if your opponent incorrectly deviates from his optimal calling range. Especially on the bubble, there are many situations where pushing a very wide range is highly +*EV* if your opponent (correctly) calls very tight. However, if your opponent is not aware of correct play, and calls significantly looser, the spot will be highly -*EV* for both of you, while all other players at the table will gain *EV*.

In these spots, be sure to check the calculated calling ranges before pushing, and evaluate if your actual opponent is calling as tight as optimal play indicates. Many poor players will have no idea about optimal play, and call much wider than optimal in these spots. It is essential that you deviate from the calculated ranges in these cases.

3.2 Ranges are balanced as a whole

If a hand is in a calculated pushing range, this does not mean pushing the hand is an unexploitable push on its own. The weaker part of the push range will rely on the protection of the top part of the range. e.g: If you decide to slowplay/trap with the top of your range, your remaining push range will probably not be sufficiently balanced and the range will become exploitable.

³Basically, any payout structure other than "Winner takes All". As a rule of thumb, the flatter the payout structure, the stronger the effect.

3.3 Deeper stacks

The push-or-fold restriction is only practical up to about 10-15BB effective stacks. If effective stacks are deeper than that, making smaller opening raises will be more profitable. Even if your stack is small enough, the calculation may still be inaccurate if other players stacks are much larger.

e.g.: 8-handed, your effective stack is 5BB as BU and the players in the blinds have 25BB each. Your calculated push range will be too loose, because it will deny the SB the option to flat call your push. The SB could call your push reasonably wide in a real game, with the option to fold if the BB shows strength. In the fictitious play calculation, the SB will always have to overpush or fold, risking his whole stack in the process. This results in a tighter SB calling range during the *Fictitious Play* calculation, and therefore in a wider pushing range for the shortstacked BU.

3.4 ICM Flaws

All equity estimations during the calculation are based on ICM. This means the resulting ranges inherit all the flaws of the ICM model. This includes pushing ranges being too tight for shortstacked UTG players, overestimated equity for midstacks on the bubble, and so on. The calculated ranges can be a good starting point for a decision, but deviation is necessary in many situations.

4 Appendix

4.1 HandRanking Charts

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	AA	KK	QQ	JJ	AKs	TT	AKo	AQs	99	AQo	AJs	88	AJo
2	ATs	77	ATo	66	A9s	55	KQs	A8s	A9o	44	KJs	A7s	KQo
3	33	A8o	A5s	A6s	KTs	22	A4s	QJs	A3s	KJo	A7o	A2s	QTs
4	JTs	K9s	A5o	A6o	KTo	T9s	J9s	Q9s	A4o	QJo	A3o	98s	K8s
5	T8s	A2o	K7s	QTo	JTo	87s	K6s	J8s	Q8s	97s	76s	K5s	K9o
6	T7s	86s	65s	K4s	T9o	J7s	Q6s	Q7s	96s	K3s	J9o	54s	Q9o
7	75s	Q5s	K2s	T6s	98o	85s	J6s	Q4s	K8o	64s	J5s	T8o	Q3s
8	95s	K7o	87o	53s	J4s	J8o	K6o	Q2s	74s	Q8o	T5s	97o	76o
9	J3s	T4s	43s	K5o	84s	63s	J2s	T7o	T3s	86o	65o	94s	K4o
10	52s	T2s	93s	J7o	73s	Q6o	96o	Q7o	92s	K3o	54o	42s	75o
11	62s	83s	Q5o	T6o	82s	K2o	32s	85o	J6o	64o	Q4o	72s	J5o
12	95o	Q3o	53o	J4o	74o	Q2o	T5o	J3o	T4o	43o	84o	63o	J2o
13	T3o	94o	52o	T2o	93o	73o	92o	42o	62o	83o	82o	32o	72o

Table 1: Push Handranking

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	AA	KK	QQ	JJ	TT	AKs	99	AKo	AQs	AJs	AQo	88	ATs
2	AJo	ATo	77	A9s	KQs	A8s	66	A9o	KJs	A7s	KQo	KTs	A8o
3	55	A6s	KJo	A5s	A7o	QJs	A4s	KTo	K9s	44	A3s	QTs	A2s
4	A6o	A5o	QJo	K8s	33	A4o	K9o	JTs	K7s	A3o	Q9s	QTo	A2o
5	K6s	22	K5s	K8o	J9s	Q8s	JTo	K4s	K7o	Q9o	T9s	K3s	Q7s
6	J8s	K2s	K6o	Q6s	T8s	K5o	J9o	Q8o	Q5s	J7s	98s	K4o	Q4s
7	T9o	Q3s	K3o	T7s	J8o	Q7o	Q2s	87s	97s	J6s	K2o	Q6o	J5s
8	T6s	T8o	J4s	76s	Q5o	86s	96s	98o	J7o	J3s	65s	Q4o	J2s
9	75s	T5s	54s	T7o	Q3o	85s	T4s	95s	87o	97o	J6o	Q2o	T3s
10	64s	J5o	74s	T2s	53s	T6o	84s	76o	94s	J4o	86o	96o	43s
11	93s	63s	J3o	92s	65o	73s	52s	J2o	83s	75o	T5o	54o	82s
12	85o	42s	95o	T4o	62s	32s	64o	T3o	72s	74o	53o	T2o	84o
13	94o	43o	93o	63o	92o	73o	52o	83o	82o	42o	62o	32o	72o

Table 2: Call/Overcall Handranking